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Planar Measurements of Droplet Velocities and Sizes Within a Simplex Atomizer

Derik C. Herpfer*
Allison Engine Company, Indianapolis, Indiana 46206-0420
and
San-Mou Jeng†
University of Cincinnati, Cincinnati, Ohio 45221-0070

Streaked particle imaging velocimetry and sizing (SPIVS), a nonintrusive laser-based optical probe for planar measurement of individual droplet velocities and sizes within a spray, is demonstrated. SPIVS, an outgrowth of particle imaging velocimetry, was developed to facilitate droplet velocity and size measurements. Inherent features of the technique are the provision of droplet image pairing and time sequencing of droplet image pairs during the image acquisition stage and measurement of droplet size from the recorded total light-scattering energy for each droplet streak. The ability of the SPIVS technique to accurately and reliably determine droplet size and velocity is evaluated by a comparison test between this probe and a phase/Doppler particle analyzer instrument. Measurements of droplet size and velocity at various radial positions within a test spray were compared. The results of this comparison study are presented along with a description of the SPIVS experimental technique.

Introduction

THE objective of this study was to evaluate the ability of the streaked particle imaging velocimetry and sizing (SPIVS) technique to make droplet size and velocity measurements within a spray. Previous work[1–4] has demonstrated and validated the overall feasibility and image processing techniques necessary to implement the SPIVS probe. The approach of this study, in demonstrating the capability of the SPIVS technique, was to compare the results generated by the SPIVS probe for an experimental test spray with those of a phase/Doppler particle analyzer (PDPA) instrument.

The SPIVS technique has been used for characterization of both burning[1] and nonburning[2–4] fuel sprays. SPIVS measurements of droplet size and velocity have been demonstrated within sprays varying from those of simple atomizers[2] to atomizers used in modern gas turbine combustors (GE SNECMA CFM56).[1,4] Very little work has been done with the particle imaging velocimetry (PIV) technique, from which SPIVS was developed, for spray and combustion measurements, although the use of the liquid droplets within a spray as the scattering particles for PIV measurement simplifies the experimental hardware needed. Farrell[5] has used a PIV technique to measure average droplet velocity and size in dilute regions of a liquid spray. Droplet size in the PIV method of Farrell was determined from the measured diameter of each droplet recorded on the photographic images. Sanz Andres[6] has used a PIV technique to measure gas velocity in the combusting environment of a laminar gas diffusion flame.

SPIVS Background

SPIVS was developed to facilitate PIV droplet velocity measurements in a liquid spray. Standard PIV measurement techniques are not always reliable in the complex flowfields of the liquid-spray systems typically found in gas turbine combustion devices. The spray flowfields of these devices are often without preferred droplet speed and direction and almost always contain a wide spectrum of droplet sizes. The SPIVS system is able to measure simultaneously the size and velocity of multiple individual droplets within a spray. The primary advantages of the SPIVS method compared to traditional PIV systems are its provision of particle image pairing and time sequencing of particle image pairs during the image acquisition stage and its measurement of droplet size from the total light-scattering energy recorded for each droplet streak.

In the SPIVS system, a single camera is used to collect the Mie scattering from two laser light sheets, which define the probe image plane, as shown in Fig. 1. One sheet is a long-pulsed (LP) sheet used to create a streaked image of a droplet's trajectory (light-pulse length on the order of 200 μs). The other is a double-pulsed (DP) sheet as used in PIV, two very short laser pulses (order of 10 ns) with a preselected time separation between pulses to freeze droplet motion and obtain distinct double images from each droplet. The two light sheets are centered one within the other with their pulse timings synchronized. The image collected from this combined LP-DP laser light sheet provides information on individual droplet velocity and size; the size is determined by the recorded total streak energy between the double image of a droplet. Two typical SPIVS images produced by this arrangement are shown in Figs. 2 and 3. Figure 2 depicts a SPIVS image of the complex droplet flowfield produced by a GE SNECMA CFM56 airblast atomizer. Figure 3 gives a SPIVS image of the relatively more simple droplet flowfield of the simplex atomizer tested in this study.

In the following two subsections, the velocimetry and drop-sizing probes of the SPIVS technique are briefly outlined. A complete analysis of the SPIVS probe, along with a thorough discussion of the system's measurement uncertainty, is provided in Ref. 7.

SPIVS Velocity Probe

Droplet velocity is determined, as in PIV, by measuring the spatial displacement of a droplet during a known time interval. In a single-exposure double light-pulse image, the spatial displacement of a droplet is recorded as an image pair, a double image of a single

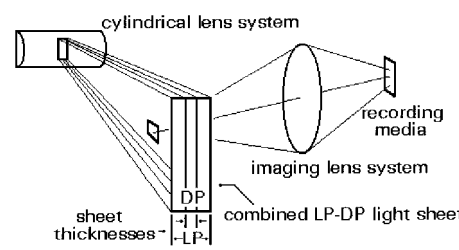

Fig. 1 Combined LP-DP light sheet arrangement.

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*Senior Project Engineer. Member AIAA.
†Associate Professor, Department of Aerospace Engineering and Engineering Mechanics. Member AIAA.

droplet with each image in an image pair corresponding to one of the two light pulses. Droplet speed is then determined for each image pair by dividing the measured displacement between each image of an image pair by the known time separation between the two light pulses that created each image pair. For the typical SPIVS images of Figs. 2 and 3, paired images are automatically identifiable by the streak between (linking) droplet double images. To obtain a droplet's velocity vector from its recorded double image (image pair), it is necessary to know the time interval and sequence of images in an image pair. The streak linking the double images in the SPIVS technique indicates the time sequence of the image pairs from its attached tail.

The image collected from each individual droplet by the SPIVS system is dependent on the droplet's path during the light-on period

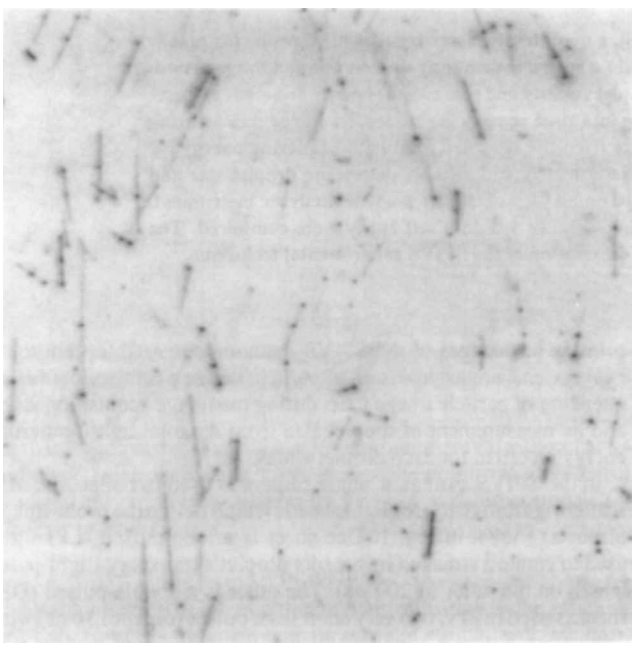

Fig. 2 SPIVS image GE SNECMA CFM56 spray (negative image, 12.5 × 12.5 mm field of view, at spray centerline 16.75 mm downstream from nozzle).

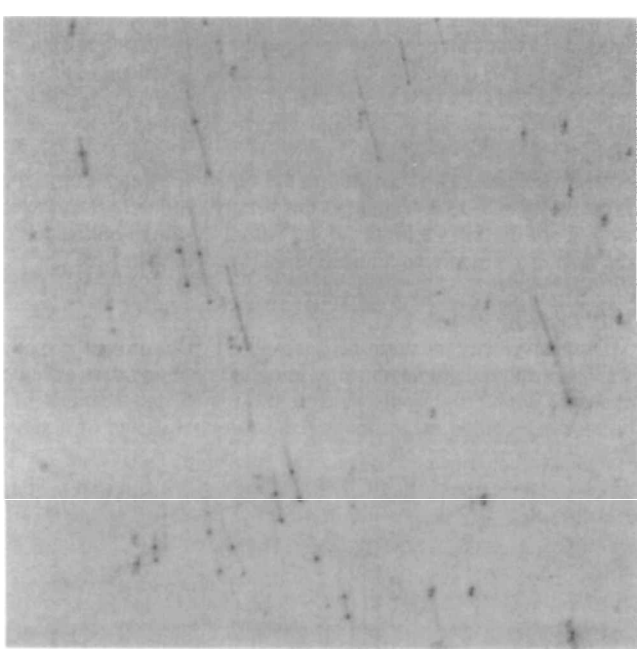

Fig. 3 SPIVS image Delavan WDB 40 psid spray midpoint (negative image, 12.5 × 12.5 mm field of view, 152.4 mm downstream from nozzle at r = 44.0 mm).

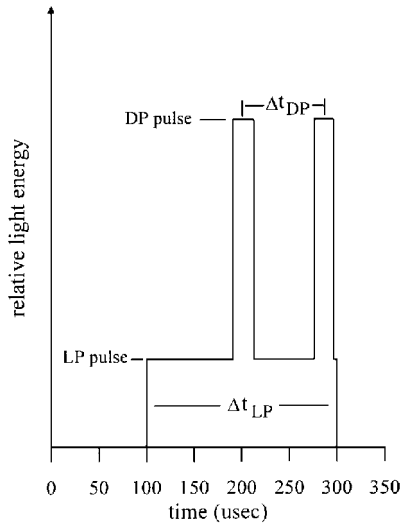

Fig. 4 LP-DP light pulse timing.

of the pulsed light sheets. For the time sequence of the synchronized DP and LP light pulses depicted in Fig. 4, a streaked droplet double image will have a tail from the LP light pulse identifying which double image from the DP light pulse came first; the tail of the streak will be created before the first image of the DP light pulse. Therefore, in this configuration, the tail trails behind the direction of droplet motion. The SPIVS system determines individual droplet velocities for all streaked droplet double images recorded on an acquired image by measuring the recorded displacement between each individual linked droplet double image and dividing by the known time separation between the DP light pulses.

Because SPIVS velocimetry and drop sizing is based on information recorded by planar images, sampling bias may occur for droplets with a large component of velocity normal to the plane of the LP-DP light sheet. The probability for a droplet with a given velocity to be sampled by SPIVS is a function of DP light-sheet thickness, time separation between light pulses, and the optical setup of the imaging system. Droplets with a high normal velocity component have less chance to be sampled by the probe than those with a low normal velocity component. The relative probability of droplets with different normal velocity components to be sampled by the SPIVS probe is

$$P = \mathrm{Max}\{[1 - |U_z|(\Delta t_{DP}/\Delta T_{DP})], 0\} \tag{1}$$

where U_z is the droplet normal velocity, Δt_{DP} is the time separation between the double light pulses, and ΔT_{DP} is the thickness of the DP light sheet. The maximum droplet normal velocity component that can be successfully sample by SPIVS is

$$|U_{z,\max}| = \Delta T_{DP}/\Delta t_{DP} \tag{2}$$

Droplets with a normal velocity component greater than $U_{z,\max}$ will not be sampled by the SPIVS probe.

The planar component of droplet velocity can also have an adverse effect on SPIVS velocimetry. In SPIVS, the minimum planar velocity that can be sampled is

$$|U_{i,\min}| = S/\Delta t_{DP} \tag{3}$$

where S is the minimum distance between droplet image pair centers that can be resolved by a given SPIVS image system configuration. S is a function of image magnification, resolution of the recording medium, and droplet size. For a given image field of view and time separation between droplet image pairs, Δt_{DP}, droplets with a high planar velocity have less probability of being successfully sampled than slower droplets. The probability for a droplet with a given planar velocity, U_i, to be sampled by SPIVS at point p within its image field of view can be stated as

$$P = \left\{\mathrm{Min}\left[\left(\frac{|x_p|}{|U_{i,x}|\Delta t_{DP}}\right), 1\right]\right\}\left\{\mathrm{Min}\left[\left(\frac{|y_p|}{|U_{i,y}|\Delta t_{DP}}\right), 1\right]\right\} \tag{4}$$

where x_p and y_p are the distance from point p to the closest edge of the image field of view in the x and y axis directions, and $U_{i,x}$ and $U_{i,y}$ are the planar components of droplet velocity. The overall probability of sampling a droplet with a given planar velocity on a SPIVS image, and therefore the sampling bias imposed by a finite field of view on the SPIVS system, can be found by integrating Eq. (4). The probability of sampling droplets with different planar velocity components on the finite size of a SPIVS image is

$$P = (\mathrm{Max}\{[1 - |U_{i,x}|(\Delta t_{\mathrm{DP}}/L_x)], 0\}) \times (\mathrm{Max}\{[1 - |U_{i,y}|(\Delta t_{\mathrm{DP}}/L_y)], 0\}) \quad (5)$$

where L_x and L_y are the length scales of the image field of view. A full review of the limitations of SPIVS velocimetry and how they pertain to actual spray measurements is provided in Ref. 7.

SPIVS Sizing Probe

In the SPIVS technique, droplet size is determined from the Mie scattering intensity collected on the images recorded for the velocity probe. For fixed collection optics 90 deg relative to the plane of the combined LP-DP light sheet, the light energy scattered from each droplet received by a camera is proportional to the total local laser light flux incident on the droplet and the droplet Mie scattering cross section integrated over the solid angle of the camera lens. The scattering cross section of a droplet is dependent on the viewing angle, the droplet size parameter, and the optical properties of the liquid.

A study based on a computational model[8] of the Lorenz–Mie scattering theory was conducted for a representative experimental SPIVS setup. For this computation, the collection optics solid angle was 0.035 sr, corresponding to the experimental setup of this study, the incident light was linearly polarized with a wavelength of 514.5 nm, and the local incident light flux on the droplet was assumed constant. Figure 5 compares the light scattered by a droplet as computed by this model to that approximated by geometric optics. The considerable scatter of the calculated Mie scattering plotted in Fig. 5 is due to the resonant nature of light scattering by a transparent particle.[9] For a given incident wavelength, small variations in a droplet's diameter may produce large variations in its scattered signal if the collection solid angle is small. Glantschnig and Chen[10] showed that, in general, the geometric optics approximation of the exact Lorenz–Mie theory is reasonable for droplets with diameters >5 μm.

In the geometric optics approximation of Mie scattering, the intensity of the scattered light is proportional to the square of droplet diameter. The curve for geometric optics shown in Fig. 5 has a proportional constant fitted to the mean scattering data of the largest droplet diameters of the Lorenz–Mie scattering calculations. All values of scattered light intensity on this figure were normalized to

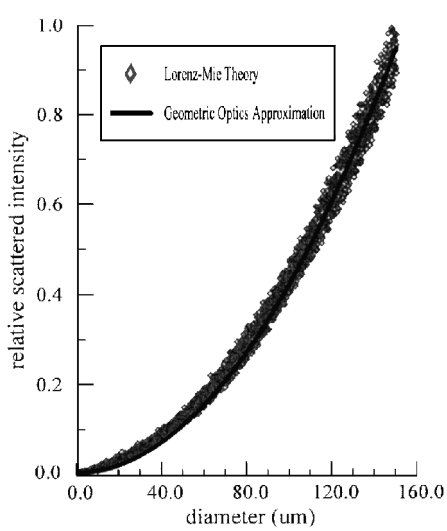

Fig. 5 Geometric optics vs Lorenz–Mie theory.

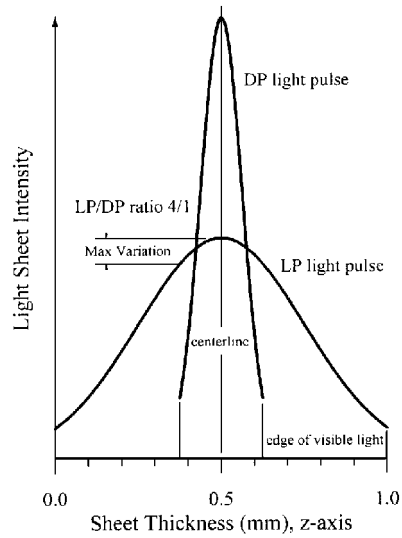

Fig. 6 LP-DP light-sheet Gaussian intensity distributions.

the largest recorded intensity value. The geometric optics approximation has an uncertainty in the determination of exact droplet size of around 10 μm for a 100-μm-diam droplet, with the uncertainty in the determined droplet diameter decreasing with decreasing drop size. Figure 5 also shows that a curve fit of the geometric optics approximation [based on Eq. (6)] of light scattering to the results of the Lorenz–Mie theory will have a slight bias depending on the proportionality constant chosen.

The SPIVS probe measures droplet size by using the geometric optics approximation to determine size from the recorded LP light-sheet Mie scattering of droplets on an image. In a single laser beam system, droplet size determination from a total scattered light energy measurement cannot be used because the laser beam intensity is not spatially uniform over the probe volume. This leads to a problem known as Gaussian ambiguity, where the local incident laser light flux on a droplet cannot be determined because the droplet spatial location within the Gaussian beam profile is not known. SPIVS resolves this problem by marking the location of each droplet within the Gaussian profile of the LP light sheet. Because the DP light sheet is thinner and centered within the LP light sheet in the SPIVS probe, the streak between the recorded double droplets of a SPIVS image must be spatially located within the center region of the LP light sheet. Therefore, the streak segment between a double image represents the light scattering from the marked center region of the LP sheet by an individual droplet during the DP light-on period, Δt_{DP}. The incident LP light intensity on a droplet within the segment of the Gaussian profile LP sheet marked by the DP light-sheet thickness is nearly constant with respect to the normal z direction (see Fig. 6). The variation of light intensity in the plane of the LP light sheet is assumed constant. This allows the incident LP light intensity to be assumed uniform on the streak segment between the double images of a SPIVS image. By integrating the intensity readings within the area of this segment, absolute droplet size can be determined from the total light energy collected, e_{tot}, by

$$e_{\mathrm{tot}} = C_{\mathrm{sys}} D_d^2 \quad (6)$$

where C_{sys} is an experimentally derived constant that takes into account collection efficiency and losses in the optical system, incident light energy, and collection light-on time scales. C_{sys} must be calibrated by using liquid droplets of known size for each experimental SPIVS configuration to determine absolute droplet size.

Experiment

Droplet size and velocity measurements were made in the spray of a Delavan WDB 45 solid-cone simplex swirl atomizer with both the SPIVS and a PDPA instrument. The Delavan spray nozzle was mounted vertically downward on a three-dimensional motorized transverse in an open area with a drain. A fan placed downstream of the spray created a downdraft to prevent splashback of droplets into

the spray from the surface of the drain. The downward air velocity at the nozzle orifice was less than 0.25 m/s. Water was used for this experiment, with tests conducted with a nozzle orifice pressure drop of 689 kPa (100 psid) and a corresponding liquid flow rate of 19.2 g/s. SPIVS and PDPA measurements were made at various radial positions within the spray at an axial distance of 152 mm (6 in.) from the nozzle. SPIVS images were collected at 11 radial positions; 50 images were recorded per location. PDPA measurements were made at four positions within each SPIVS location's field of view (12.5 × 12.5 mm). The overall test procedure was modeled on the American Society for Testing Materials round-robin tests of 1985 (Ref. 11) in which various drop sizing instruments were tested and compared.

The PDPA instrument was a modified aerometrics two-component system using the laser delivery optics of a Dantec laser Doppler velocimetry system. A multiline coherent argon-ion laser was the laser source for the PDPA system. This laser, tuned to 514.5 nm, was used as the light source of the LP sheet for the SPIVS probe. A Quantel DP neodymium:yttrium-aluminum garnet laser was the light source of the DP sheet. For SPIVS, the light of both of these lasers was formed into sheets by using a light train of four cylindrical lenses for each laser beam. The two sheets were combined by stacking their respective lens systems one atop the other with the top system tilted downward so that the light sheets overlapped. The LP-to-DP light-sheet thickness ratio was set at 4:1. With this setup, the total uncertainty of a droplet size measurement ($d^2 = e_{tot}/C_{sys}$), considering the optical arrangement and the accuracy of the geometric optics approximation, is around 20%. The imaging system of the SPIVS instrument consisted of an EG&G 18-bit intensified 512 × 512 charge-coupled device camera with a Nikon f/2.8 macrolens. The camera field of view was set at 12.5 × 12.5 mm and the LP light-sheet pulse length was controlled by the camera's exposure time. The SPIVS instrument was calibrated by using three different water drop sizes produced by an Aerometrics MDG-100 monodisperse drop generator.

A detailed description of the SPIVS system setup of this study, as well as a full discussion on the overall experimental procedure and the experimental results, is given in Ref. 7.

Experimental Results

The objective of this study was to evaluate the measurement by SPIVS of droplet velocity and absolute size within a liquid spray. SPIVS droplet size and velocity measurements were compared to those of a PDPA instrument for the same test spray. For comparison of SPIVS and PDPA in this study, the results from these two instruments were converted to the same format. The PDPA instrument[12] is a point measurement single-particle counting technique where the average drop sizes and velocities recorded are based on a temporal weighting of mean quantities. The SPIVS system collects the size and velocity of many droplets within its probe volume, a combination of the camera field of view and the DP light-sheet thickness, at a given instant of time. SPIVS therefore produces results based on a spatial weighted sample. A comparison between the two different diagnostic probes requires the mean quantities measured by both instruments to be based on the same sample weighting. The commercial PDPA software includes a conversion process[13] that enables a temporal weighted sample to be changed into a spatial weighted sample. For direct comparison between the drop sizing results of the SPIVS and PDPA techniques, computed mean drop sizes for both instruments are based on a spatial weighted sample.

Figures 7–9 give the drop size distributions (histograms of droplet size) measured by both the PDPA and SPIVS instruments at three representative radial locations within the spray ($r = 0$, 42, and 84 mm, respectively) for the test 152-mm (6-in.) axial distance from the nozzle. The PDPA drop size histograms in these figures have been weighted to a spatial sampling average. In Fig. 7, the drop size distributions measured by SPIVS and PDPA are almost identical. For the outer regions of the spray ($r = 42$ and 84 mm; Figs. 8 and 9, respectively), the size distributions recorded by both probes are similar. SPIVS measured a larger percentage of small droplets (range, 40–60 μm) than PDPA in the regions represented by Figs. 8 and 9. However, SPIVS did detect a few very large droplets (>250 μm), in these regions that PDPA did not. Because of this, the mean drop sizes measured [Sauter mean diameter (SMD)] by SPIVS closely follow the PDPA results in the outer regions of the spray, as shown

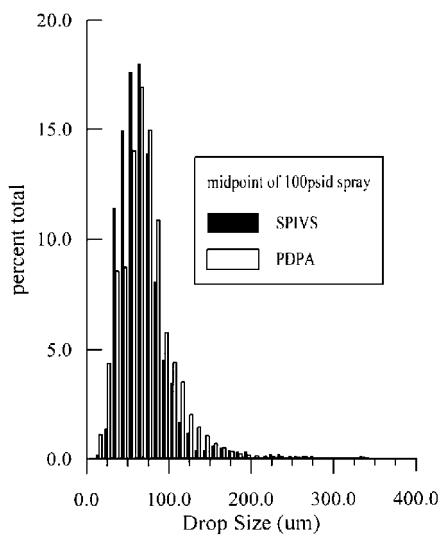

Fig. 8 Drop size distributions at r = 42 mm.

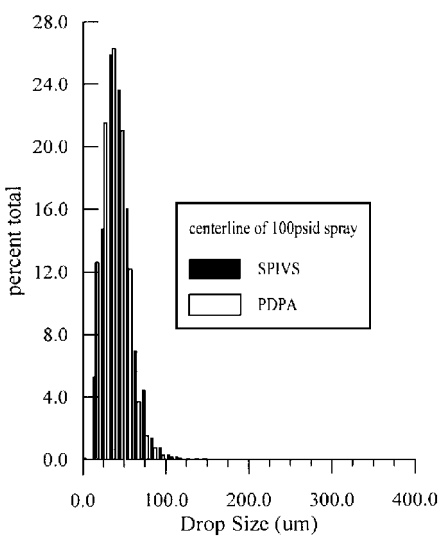

Fig. 7 Drop size distributions at r = 0.

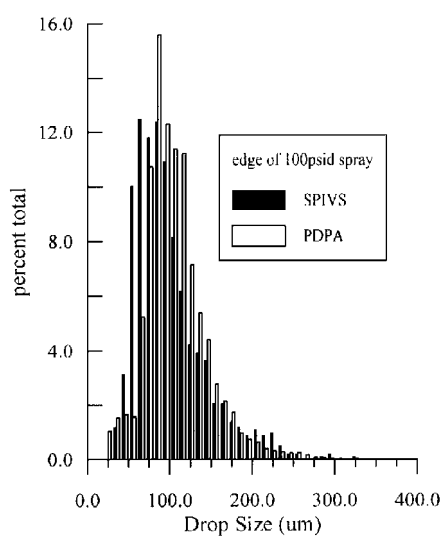

Fig. 9 Drop size distributions at r = 84 mm.

in Fig. 10, although the drop size distributions that the mean drop sizes were based on were slightly different. The SPIVS distributions have peaks at a smaller drop size than the PDPA distributions. In all regions of the spray (Figs. 7–9), the SPIVS probe measured fewer very small droplets than the PDPA instrument for the same spray because of the sensitivity of the digital imaging system.

In Figs. 7–9, the SPIVS results at each radial location were based on a significantly smaller sample size than those for the PDPA instrument. SPIVS histograms were based on a sample size of approximately 1000 droplets, whereas the PDPA histograms were based on the data of 12,000 samples. Therefore, the uncertainty of the SPIVS histograms in Figs. 7–9 accurately depicting the true droplet number distribution is greater than that for the PDPA results. The statistical uncertainty of a measurement, determined from a sample size N, is proportional to $N^{-1/2}$. For a histogram, the height of each bar has an uncertainty based on the number of samples per bar. For the SPIVS histograms, a bar indicating 1% of the total sample would be based on roughly 10 droplets, with a resultant uncertainty of the recorded value of roughly 30%. For PDPA, 1% of the total sample, or 120 droplets, would give an uncertainty to the histogram bar of 9%. A 10% uncertainty for the value of a histogram bar would require a sample size of roughly 100 droplets, or, for SPIVS, 10% of the total data recorded for a histogram plot. Because of the small sample sizes of the SPIVS probe, SPIVS histograms in Figs. 7–9 have uncertainties on the order of 30% for all bar levels less than 1%.

Figure 10 shows the radial variation of the SMD measured by the SPIVS and PDPA instruments. Both the spatial and temporal weighted averages of the PDPA results are presented along with the SPIVS data. The temporal PDPA results are included only as a reference; the comparison of SMD is made between the SPIVS data and the spatial PDPA values. The SPIVS results show a close agreement with the PDPA measurements, slightly overestimating the SMD at around $r = 65$ mm. The average deviation of SMD measurements between the two test instruments over the radial profile was 10 μm. The drop sizing results of both instruments showed that the spray was symmetric around its centerline, as expected in an axisymmetric spray.

Figures 11–13 give the mean axial velocity as a function of drop size measured by both SPIVS and PDPA at the representative spray locations ($r = 0$, 42, and 84 mm). The zero values in these figures indicate that no drops of that size were recorded. Figure 11 indicates that the mean axial velocity for each size group recorded by both instruments at the $r = 0$ location was very close until about 100 μm. Above this size, SPIVS did not detect the presence of any droplets, whereas PDPA registered some droplets up to 400 μm, with less than 1% of the total PDPA samples having sizes greater than 100 μm. At $r = 42$ mm (Fig. 12), SPIVS results follow the PDPA data closely in the drop size range 0–200 μm. At $r = 84$ mm (Fig. 13), the SPIVS data are only an approximate fit to PDPA in the drop size range 0–250 μm. In all of these figures, the axial velocity vs drop size measured by both instruments is in close agreement up until a certain value of drop size (the larger drop sizes). The results between PDPA and SPIVS for the larger drop sizes exhibit greater fluctuations because of the very small statistical number of samples

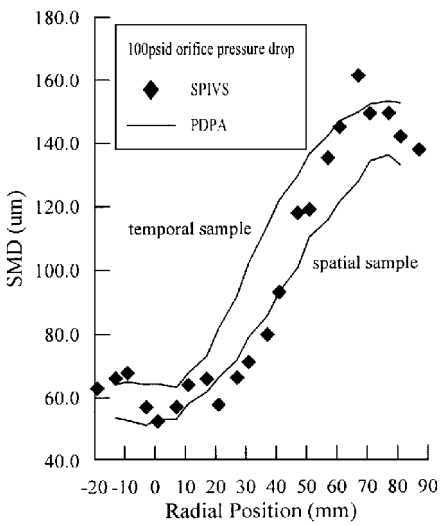

Fig. 10 Comparison of SMD in spray.

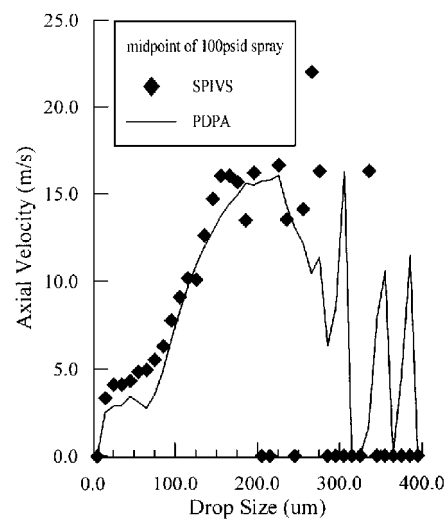

Fig. 12 Axial velocity vs drop size at $r = 42$ mm.

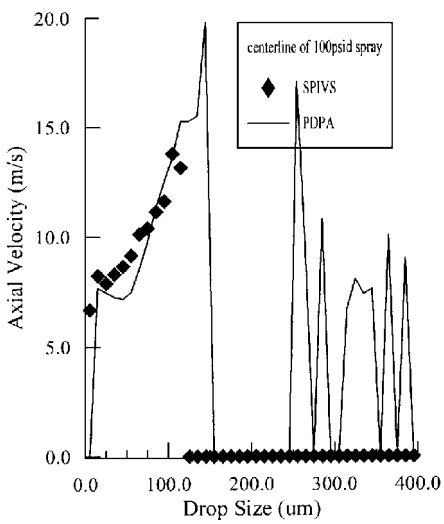

Fig. 11 Axial velocity vs drop size at $r = 0$.

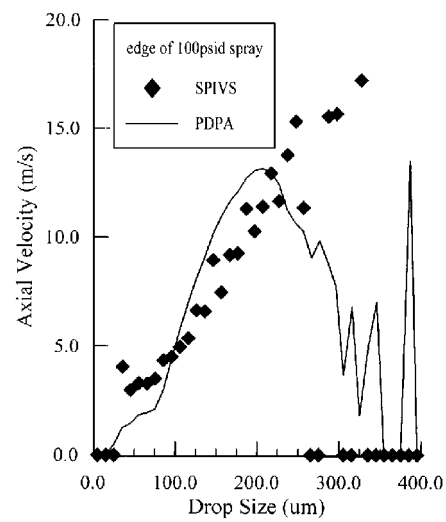

Fig. 13 Axial velocity vs drop size at $r = 84$ mm.

collected for the larger droplet sizes. For example, the size range 0–150 μm at the $r = 42$ mm measurement location corresponds to 93% of the total recorded SPIVS data and 95% of the PDPA total data at this location. The statistical uncertainty in the data for the larger droplet sizes in this study prevents the accurate correlation of drop size to axial velocity for the larger drop sizes. In general, the results of this study indicate that the average axial velocity of a droplet increases with increasing drop size. This is expected, because in the downstream regions of a spray the momentum of a large droplet will be closer to the initial value imparted by the liquid injector than the momentum of a small droplet.

Sampling bias due to the system limitations of SPIVS velocimetry (i.e., out-of-plane motion of droplets) was not significant in the sprays used in this study. An analysis of the measurement uncertainties affecting the SPIVS technique as well as a detailed discussion of the experimental results presented in this study is provided in Ref. 7

Conclusions

SPIVS has been demonstrated for nonintrusive measurement of the size and velocity of droplets in the spray of a Delavan WDB 45 solid-cone simplex swirl atomizer. The ability of the SPIVS technique to accurately and reliably determine these spray characteristics was evaluated in this study by a comparison test of the SPIVS probe with a PDPA instrument.

The results of the comparison study indicate that the SPIVS diagnostic technique has the ability to accurately and reliably characterize the liquid droplet properties within a spray. The measurement results in this study indicate that SPIVS drop sizing can resolve mean sizes in close agreement with those of the PDPA spray diagnostic technique. The SPIVS system, however, does exhibit a tendency toward sampling bias in favor of the larger drop sizes within a spray. A larger droplet is more likely to be detected and therefore sampled in SPIVS than a smaller-sized droplet because of the sensitivity of the digital imaging system. The SPIVS velocimetry results showed the overall ability of the system to measure droplet mean axial velocities in conjunction with drop size.

A detailed review of the comparison tests conducted for this research study, which is provided in Ref. 7, indicated that the total measurement uncertainty in SPIVS drop sizing of an individual droplet was 16%. The uncertainty in an individual droplet's velocity for SPIVS velocimetry was 7%.

Acknowledgments

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